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Controlling viscoelastic flow by tuning frequency during occlusions

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We study the flow of a viscoelastic fluid flowing in an occluded tube due to either central or peripheral obstructions. We show that, by driving the fluid with a dynamic pressure gradient at the frequency that maximizes the dynamic permeability of the obstructed system, the magnitude of the flow can partially be recovered without the removal of the obstruction. We compare the results obtained for the two types of occlusions studied and find that flow recovering is larger in the case of central occlusions.

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I. INTRODUCTION

Occlusions of tubes have always represented a problem. From engines and filters to arteries and bronchia we can find a countless amount of systems in which the lack of movement of a fluid due to the presence of an obstacle results in the partial or total failure of a process. In particular, the occlusion of biotubes in the human body represents an important issue in many diseases. For instance, during the occlusion of arteries, blood decreases its velocity and, in critical cases, is effectively unable to flow through the remaining space. Such a lack of movement prevents irrigation and, in many cases, results in the eventual death of tissues.

Recent experimental and theoretical work on viscoelastic fluids [1–6] have found that the dynamic permeability can increase orders of magnitude at certain frequencies. The dynamic permeability is an intrinsic property of the system viscoelastic fluid-confining media and determines the system response to different signals of the pressure gradient. It can be considered as a measure of the resistance to flow, the larger the dynamic permeability, the less the resistance to flow. The increase of the dynamic permeability at certain frequencies suggests that the magnitude of the flow might be increased by driving the fluid with a pressure gradient that contains the frequency that maximizes the dynamic permeability. We have indeed verified that by imposing a periodic pressure gradient at the frequency that maximizes the dynamic permeability, the magnitude of the flow of a viscoelastic fluid flowing in a tube can largely be increased. This implies that the dynamics for the pressure gradient with a properly chosen frequency provides a way of controlling the magnitude of the flow. We present analytical results for the simple case of a pressure gradient consisting of a single sinusoidal mode in order to show that the maximum value of the flow magnitude depends on two things: the real part of the dynamic permeability and the cross-sectional area available for flow. When an obstruction occurs, it is clear that if one recovers the value of the real part of the dynamic permeability (by driving the fluid at the proper frequency), one eliminates one of the two factors that provoke the dramatic decrease of flow. Having established that, we model two types of occlusions and show that, by driving the fluid with a periodic pressure gradient at the frequency that maximizes the permeability of the obstructed system, the flow can partially be recovered without the removal of the obstruction. We compare the results obtained for the two types of occlusions studied, namely central occlusions and peripheral occlusions and find that flow recovering is larger in the case of central occlusions. We also compare the results for two different dynamics of the pressure gradient and find that even though the dynamics does make a difference in the magnitude of the flow, it does not make a big difference when it comes to the percentage of flow that can be recovered.

II. MODEL

We model the viscoelastic fluid, by means of the linearized Maxwell model, which is the simplest hydrodynamic model to describe viscoelastic behavior, i.e.,

$$t_r\rho\frac{\partial^2\mathbf{v}}{\partial t^2}+\rho\frac{\partial\mathbf{v}}{\partial t}=-t_r\frac{\partial\nabla p}{\partial t}-\nabla p+\eta\nabla^2\mathbf{v}. \quad (1)$$

Here t is time, $\mathbf{v}$ is velocity, p is pressure, and t_r, ρ, and η are the Maxwell relaxation time, density, and viscosity of the viscoelastic fluid. Equation (1) is nothing but the linearized momentum balance equation of hydrodynamics together with the constitutive relation of the Maxwell model. This one is built in such a way that in the limit $t_r\to 0$ reduces to the constitutive equation of a Newtonian fluid, and in the limit $t_r\to\infty$ reduces to the constitutive equation of an elastic solid. The Maxwell relaxation time is defined as $t_r\equiv\eta/G$ where G is the elastic modulus of the fluid. Equation (1) should be solved for a particular geometry which in turn depends on the type of occlusion considered. We consider occlusions that result from the partial obstruction of flow in two different ways. The first type of obstruction is one in which the walls of a tube have been internally engrossed and the fluid circulates through a tube that has effectively a smaller radius. We call this *peripheral occlusion* and we model the space of flow as the one inside a cylinder with a radius smaller than the one of the unobstructed tube. The second type of obstruction considered is one in which the fluid must flow between

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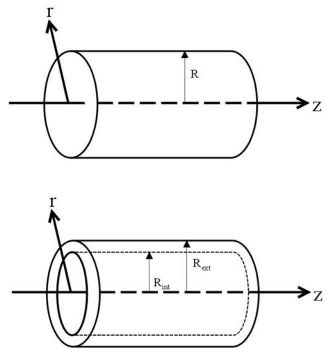

FIG. 1. The top figure shows the geometry considered for peripheral occlusions while the bottom figure shows the geometry considered for central occlusions.

the walls of a tube and an obstacle inside it. We call this *central occlusion* and we model the space of flow as the one between two concentric cylinders. We solve Eq. (1) for these two geometries that are illustrated in Fig. 1.

In both cases, we solve Eq. (1) in the frequency domain subject to no-slip boundary conditions. We then average the velocity over the cross-sectional area available for flow and obtain a generalized Darcy's law, i.e.,

$$\langle \hat{v}_z \rangle = -\frac{\hat{K}(\omega)}{\eta} \nabla \hat{p}, \tag{2}$$

where $\hat{K}$ is the dynamic permeability in the frequency domain. Here $\hat{v}_z$ and $\nabla \hat{p}$ are the velocity and the pressure gradient in the frequency domain, that is, they are functions of space and frequency.

For the cylindrical geometry, that is the geometry used to simulate both, unobstructed tubes and peripheral occlusions, the dynamic permeability is given by [1,2]

$$\hat{K}(\omega) = -\frac{\eta}{i\omega\rho}\left[1 - \frac{\langle J_0(Ar)\rangle}{J_0(AR)}\right], \tag{3}$$

where r is the radial coordinate, R is the radius of the cylinder, ω is the frequency, $i=\sqrt{-1}$, J_0 is the Bessel function of order zero, $A=\frac{\rho}{\eta}(t_r\omega^2+i\omega)$ and the average of the Bessel function refers to the average over the cross section and is given by

$$\langle J_0(Ar)\rangle = \frac{2J_1(AR)}{AR}. \tag{4}$$

For the geometry of concentric cylinders, that is, the one used to simulate central occlusions, we obtain

$$\hat{K}(\omega) = -\frac{\eta}{i\omega\rho}\left[1 - \frac{N_0(AR_{\text{int}}) - N_0(AR_{\text{ext}})}{F}\langle J_0(Ar)\rangle + \frac{J_0(AR_{\text{int}}) - J_0(AR_{\text{ext}})}{F}\langle N_0(Ar)\rangle\right], \tag{5}$$

where R_{ext} and R_{int} are the radii of the outer and inner cylinders, respectively, N_0 is the Neumann function of order zero, $F=J_0(AR_{\text{ext}})N_0(AR_{\text{int}})-J_0(AR_{\text{int}})N_0(AR_{\text{ext}})$ and the averages of the Bessel and Neumann functions refer to averages over the cross section available for flow and are given by

$$\langle J_0(Ar)\rangle = \frac{2[R_{\text{ext}}J_1(AR_{\text{ext}}) - R_{\text{int}}J_1(AR_{\text{int}})]}{A(R_{\text{ext}}^2 - R_{\text{int}}^2)} \tag{6}$$

and

$$\langle N_0(Ar)\rangle = \frac{2[R_{\text{ext}}N_1(AR_{\text{ext}}) - R_{\text{int}}N_1(AR_{\text{int}})]}{A(R_{\text{ext}}^2 - R_{\text{int}}^2)}. \tag{7}$$

The flow as a function of time, $Q(t)$, is given by

$$Q(t) \equiv \langle v(t)\rangle \mathcal{A}, \tag{8}$$

where $\mathcal{A}$ is the cross-sectional area available for flow and $\langle v(t)\rangle$ is the average over the unobstructed cross-sectional area of the velocity in the time domain. In order to compute it, we need to impose a time-dependent pressure gradient $\nabla p(t)$, Fourier transform it to obtain a frequency-dependent pressure gradient $\nabla \hat{p}(\omega)$, we then should use Darcy's law in the frequency domain in order to obtain the velocity $\hat{v}(\omega)$ in the frequency domain, finally with the aid of an inverse Fourier transform, we should get the velocity $v(t)$ in the time domain and compute the flow.

III. DYNAMIC PERMEABILITIES

We have studied a system with the following parameters: viscosity $\eta=5.5\times10^{-3}$ kg/(m s), density $\rho=1050$ kg/m^3, relaxation time $t_r=0.5$ s, and unobstructed radius $R=1$ mm. We have considered obstructions of 75% of the cross-sectional area.

In Fig. 2, we have plotted the dynamic permeability of a nonobstructed tube and of a tube with a peripheral occlusion. Several things can be observed from the figure. First, the real part of the dynamic permeability has peaks at certain frequencies and its value at the first peak is much larger than its value at zero, or very small, frequency. We call the frequency at which the first maximum occurs, the resonance frequency of the system. Second, at low frequencies, the real part of the dynamic permeability in a tube with a peripheral obstruction is much smaller that the real part of the dynamic permeability in an unobstructed tube. Third, the first peak of each of the two curves have the same value of the real part of the dynamic permeability. That is, the real part of the dynamic permeability at the resonance frequency in a peripheral obstruction is as large as the real part of the dynamic permeability at the resonance frequency with no obstruction. Finally, we can see that for the occluded system the resonance frequency is larger than for the system with no occlusion. For the parameters chosen, the resonance frequency for the unobstructed tube is equal to $\omega_1=7.66$ rad/s and the resonance frequency for the peripheral occlusion is equal to $\omega_2=15.5$ rad/s. It is worth noticing that the imaginary part of the dynamic permeability is zero at the resonance frequency and that it goes to zero as frequency goes to zero.

In Fig. 3, we have plotted the real part of the dynamic permeability of a nonobstructed tube and of a tube with a

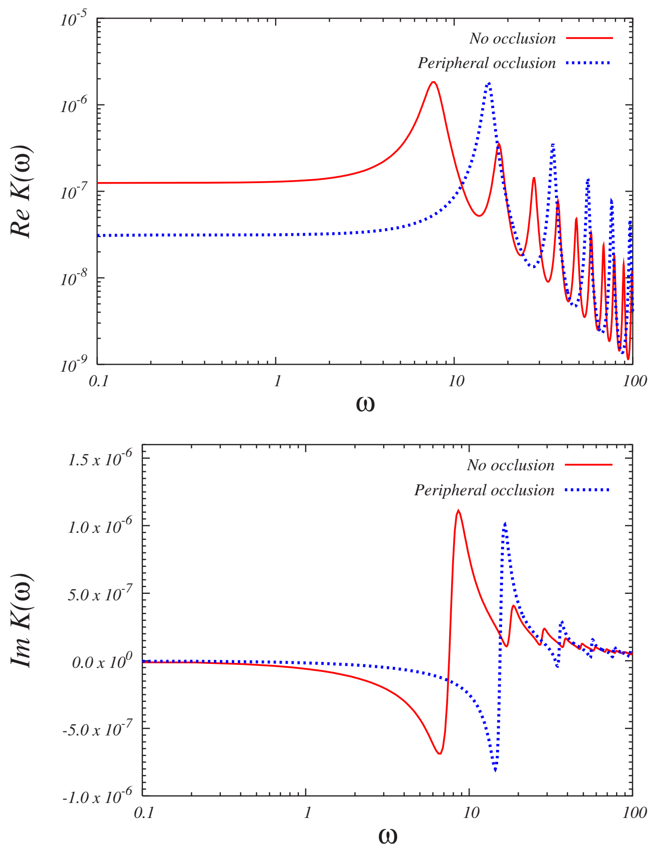

FIG. 2. (Color online) Dynamic permeability of a fluid flowing in a tube with peripheral occlusion and with no occlusion. In the peripheral occlusion, the obstruction represents 75% of the cross-sectional area. The resonance frequency of the occluded system is larger than the resonance frequency of the nonoccluded system. The dynamic permeability is given in m^2 and the angular frequency in rad/s. The parameters of the plot are described in the text.

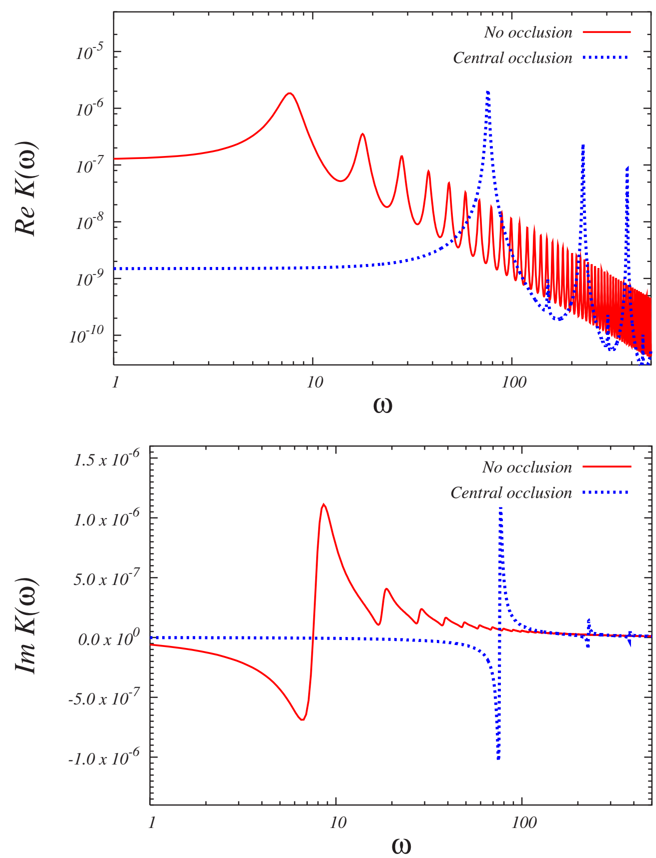

FIG. 3. (Color online) Dynamic permeability of a fluid flowing in a tube with central occlusion and with no occlusion. In the central occlusion, the obstruction represents 75% of the cross-sectional area. The resonance frequency of the occluded system is larger than the resonance frequency of the nonoccluded system. The dynamic permeability is given in m^2 and the angular frequency in rad/s. The parameters of the plot are described in the text.

central occlusion. Several things can be observed from the figure. First, at low frequencies, the real part of the dynamic permeability of the fluid with a central occlusion is much smaller that the real part of the dynamic permeability in an unobstructed tube. Second, the real part of the dynamic permeability at the resonance frequency in a central occlusion is as large as the real part of the dynamic permeability at the resonance frequency with no obstruction. That is, the first peak of the two curves shown in the figure have the same value of the real part of the dynamic permeability. Third, we can see that for the occluded system the resonance frequency is larger than for the system with no occlusion. Finally, we observe the presence of secondary peaks for the dynamic permeability in a central occlusion [7]. These are related to the presence of the inner cylinder but have no relevance for what will be discussed here. For the parameters chosen, the resonance frequency for the central occlusion is equal to $\omega_2 = 75.82$ rad/s. Just as in the case of the peripheral occlusion, the imaginary part of the dynamic permeability for the central occlusion is zero at the resonance frequency, and goes to zero as frequency goes to zero.

IV. FLOW FOR A SINGLE MODE PRESSURE GRADIENT

In order to have a physical insight of the origin of the increase in the magnitude of the flow, we first compute it for a pressure signal consisting of a single sinusoidal mode of the form

$$\nabla p(t) = \nabla p_0 \cos \omega_0 t. \tag{9}$$

This pressure gradient oscillates in time around zero and gives rise to zero net flow. By using the definition of a δ function [8] given by

$$\delta(\omega - \omega_0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} e^{i(\omega-\omega_0)t} dt, \tag{10}$$

we write the pressure gradient on the frequency domain as

$$\nabla \hat{p}(\omega) = \sqrt{\frac{\pi}{2}}\, \nabla p_0[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]. \tag{11}$$

When we substitute this in the generalized Darcy's law [Eq. (2)], we obtain the velocity in frequency domain as

$$\hat{v}(\omega) = -\sqrt{\frac{\pi}{2}}\frac{\hat{K}(\omega)}{\eta}\nabla p_0[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)], \quad (12)$$

finally, by an inverse Fourier transformation, we obtain the velocity in time domain as

$$v(t) = -\frac{\nabla p_0}{2\eta}[\hat{K}(\omega_0)e^{-i\omega_0 t}+\hat{K}(-\omega_0)e^{i\omega_0 t}]. \quad (13)$$

Since for a real time-dependent pressure gradient, the time-dependent velocity should be real as well, the dynamic permeability should be such that $\hat{K}(-\omega)=\hat{K}^{\star}(\omega)$, where the star indicates the complex conjugate of the function. Using this, we can write the velocity in time domain as

$$v(t) = -\frac{\nabla p_0}{\eta}[\text{Re}\,\hat{K}(\omega_0)\cos(\omega_0 t)+\text{Im}\,\hat{K}(\omega_0)\sin(\omega_0 t)]. \quad (14)$$

This is the velocity in time domain for a dynamic pressure signal consisting only of one mode. Now if this mode with frequency ω_0 has the resonance frequency of the system, Re $\hat{K}(\omega_0)$ takes the maximum possible value of the permeability and Im $\hat{K}(\omega_0)=0$. In this case the flow magnitude is given by

$$|Q(t)| = \frac{\nabla p_0}{\eta}\text{Re}\,\hat{K}(\omega_0)\mathcal{A}|\cos(\omega_0 t)|, \quad (15)$$

and its maximum value is given by

$$|Q(t)|_{\max} = \frac{\nabla p_0}{\eta}\text{Re}\,\hat{K}(\omega_0)\mathcal{A}. \quad (16)$$

For this simple example, it becomes obvious that the maximum value of the flow magnitude depends on two things: the real part of the dynamic permeability and the cross-sectional area available for flow. So, when an obstruction occurs in a system flowing at the resonance frequency of the unobstructed system ω_1, the flow decrease is due not only to the reduction of the cross-sectional area available for flow, but also due to the dramatic decrease on the permeability at that frequency (see Fig. 2 or Fig. 3 and compare the maximum value of the real part of the permeability of the unobstructed system to the value of the dynamic permeability of the obstructed system at the same frequency). Now, what we can see from the dynamic permeability of the obstructed system is that there is a new frequency that makes it maximum. So, according to Eq. (16) if we impose a one-mode pressure gradient at this new resonance frequency, we are restoring one of the two factors that decrease the flow magnitude, i.e., even if we cannot change the value of the cross-sectional area available for flow $\mathcal{A}$, we can restore the value for the dynamic permeability by imposing a dynamic pressure gradient with the new resonance frequency (recall that the real part of the dynamic permeability for both the unobstructed and the obstructed systems have the same value for its respective resonance frequencies).

If instead of the cosine pressure gradient we impose a one-mode sine pressure gradient of the form

$$\nabla p(t) = \nabla p_0 \sin \omega_0 t, \quad (17)$$

we obtain a velocity in time domain given by

$$v(t) = -\frac{\nabla p_0}{\eta}[\text{Re}\,\hat{K}(\omega_0)\sin(\omega_0 t)-\text{Im}\,\hat{K}(\omega_0)\cos(\omega_0 t)], \quad (18)$$

and exactly the same expression for the maximum value of the flow magnitude [Eq. (16)].

V. FLOW FOR A PERIODIC PRESSURE GRADIENT

For an arbitrary dynamic pressure gradient the Fourier transform necessary to obtain the pressure in the frequency domain and the inverse Fourier transform necessary to obtain the velocity in the time domain would need to be done with numerical methods. However, we have found that any periodic pressure gradient leads to an analytical expression for $\langle v(t)\rangle$. This is because a periodic pressure gradient can always be written as a Fourier series, i.e.,

$$\nabla p(t) = \frac{a_0}{2}+\sum_{n=1}^{\infty}a_n\cos n\omega t+\sum_{n=1}^{\infty}b_n\sin n\omega t. \quad (19)$$

Clearly, with the use of Eqs. (14) and (18) we can write

$$v(t) = -\frac{1}{\eta}\Bigg[\frac{a_0}{2}\text{Re}\,\hat{K}(0)+\sum_{n=1}^{\infty}a_n\,\text{Re}\,\hat{K}(n\omega)\cos(n\omega t) + \sum_{n=1}^{\infty}a_n\,\text{Im}\,\hat{K}(n\omega)\sin(n\omega t)+\sum_{n=1}^{\infty}b_n\,\text{Re}\,\hat{K}(n\omega)\sin(n\omega t) - \sum_{n=1}^{\infty}b_n\,\text{Im}\,\hat{K}(n\omega)\cos(n\omega t)\Bigg], \quad (20)$$

where each of the coefficients a_0, a_n, and b_n is given by the nonorthogonal part of the pressure gradient to the corresponding Fourier mode. This result allows one to determine in an analytical form the velocity in time domain for any periodic pressure signal. Obviously this expression involves an infinite sum that in turn should be evaluated numerically.

VI. FLOW FOR A PARTICULAR DYNAMIC PRESSURE SIGNAL

In order to see how sensitive the results are to particular signals of the pressure gradient, we have studied two different dynamics. Namely, a train of Gaussian pulses and a rectified sinusoidal signal. Since the results that we have obtained are very similar for both signals, we present the results for the rectified sinusoidal signal and comment on the ones for a train of Gaussian pulses. So our pressure gradient is given by

$$\nabla p(t) = \frac{\pi\nabla p_0}{2}|\sin(\omega_0 t)|. \quad (21)$$

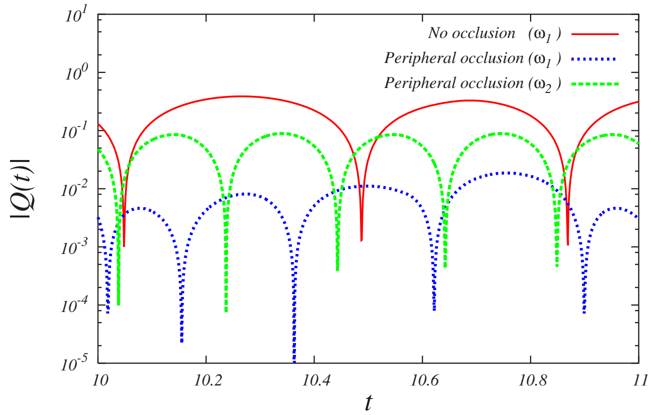

FIG. 4. (Color online) Flow magnitude vs time in a 75% peripherally occluded tube and in a nonoccluded tube. The flow magnitude is plotted for three cases: normal conditions with ω_1, obstructed tube with ω_1, and the same peripherally occluded tube but with ω_2 as described in the text. Flow is given in cm^3/s and time is given in s.

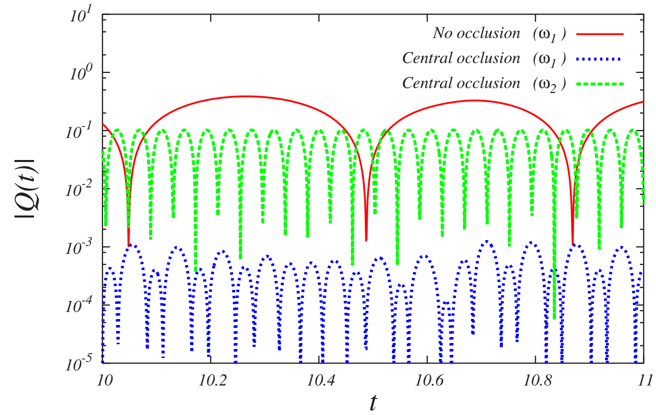

FIG. 5. (Color online) Flow magnitude vs time in a 75% centrally occluded system and in a nonoccluded system. Flow magnitude is plotted for three cases: normal conditions with ω_1, occluded system with ω_1, and the same centrally occluded system but with ω_2 as described in the text. Flow is given in cm^3/s and time is given in s.

The factor $\frac{\pi}{2}$ is introduced in order to impose a pressure gradient with time average equal to ∇p_0. Note that since the sine is rectified, the frequency of this pressure gradient is equal to $2\omega_0$; so, if one wishes to impose a particular frequency ω_p, the parameter ω_0 should be equal to $\omega_0=\omega_p/2$. The velocity of the fluid in time domain in response to such a pressure gradient is given by

$$\langle v(t)\rangle = -\frac{2\,\nabla p_0}{\eta}\left[\frac{1}{2}\mathrm{Re}\,\hat{K}(0) - \sum_{n=2,4,\ldots}^{\infty}\frac{\mathrm{Re}\,\hat{K}(n\omega_0)\cos(n\omega_0 t)}{n^2-1} - \sum_{n=2,4,\ldots}^{\infty}\frac{\mathrm{Im}\,\hat{K}(n\omega_0)\sin(n\omega_0 t)}{n^2-1}\right]. \quad (22)$$

In what follows we compute the magnitude of the flow as a function of time in nonobstructed tubes and in tubes with both peripheral and central occlusions. In all cases the net flow oscillates around its average value $Q=-\frac{\nabla p_0}{\eta}\mathrm{Re}\,\hat{K}(0)\mathcal{A}$.

We have imposed a pressure gradient with a time average equal to $\nabla p_0=-500$ Pa/m. We have approximated the sums in Eq. (22) by the first 1000 terms. This is a good approximation since for large values of n the terms decay like $\frac{1}{n^2}$.

A. Peripheral occlusions

We compare the flow in a nonoccluded system at the resonance frequency of the nonoccluded system, ω_1, with the flow in a peripherally occluded system at two different frequencies, namely, the resonance frequency of the nonoccluded system, ω_1, and the resonance frequency of the peripherally occluded system, ω_2.

The flow comparison for a peripheral obstruction is shown in Fig. 4. We can observe several points from Fig. 4. (1) The maximum value of the flow in nonoccluded tubes driven by a time-dependent pressure gradient given by Eq. (21) with $\omega_0=\omega_1/2$ is $0.3873\ \mathrm{cm}^3/\mathrm{s}$. (2) The maximum value of the flow in a peripherally occluded tube driven by exactly the same pressure gradient with $\omega_0=\omega_1/2$ is $0.0186\ \mathrm{cm}^3/\mathrm{s}$. This means that the maximum value of the flow during a peripheral occlusion decreases by 95% for an occlusion of 75% of the cross-sectional area. (3) When a pressure gradient of the form given by Eq. (21) with the new resonance frequency $2\omega_0=\omega_2$ is imposed, the maximum value of the flow increases to $0.0889\ \mathrm{cm}^3/\mathrm{s}$. This implies a recovery of 18% of the flow.

For a train of Gaussian pulses the maximum value of the flow in nonoccluded tubes driven by a time-dependent pressure gradient with ω_1 is $0.1273\ \mathrm{cm}^3/\mathrm{s}$; the maximum value of the flow in a peripherally occluded tube driven by exactly the same pressure gradient is $0.0039\ \mathrm{cm}^3/\mathrm{s}$. This means that the maximum value of the flow during a peripheral occlusion decreases by 97% for an occlusion of 75% of the cross-sectional area; finally, when a pressure gradient with the new resonance frequency ω_2 is imposed, the maximum value of the flow increases to $0.0243\ \mathrm{cm}^3/\mathrm{s}$. This implies a recovery of 16% of the flow.

We can observe two things: that for a rectified sine signal the maximum value of the flow is larger in all cases, and that when it comes to the percent of flow that can be recovered, the dynamics does not seem to make a big difference.

B. Central occlusions

Similar results are obtained for central occlusions: we compare the flow in a nonoccluded system at the resonance frequency of the nonoccluded system, ω_1, with the flow in a centrally occluded system at two different frequencies, namely, the resonance frequency of the nonoccluded system, ω_1, and the resonance frequency of the centrally occluded system, ω_2.

The flow comparison for a central obstruction is shown in Fig. 5.

We can observe several points from Fig. 5: (1) The maximum value of the flow in a nonoccluded tube driven by a time-dependent pressure gradient of the form of Eq. (21)

with $\omega_0 = \omega_1/2$ is 0.3873 cm^3/s. (2) The maximum value of the flow in a centrally occluded tube driven by exactly the same pressure gradient with $\omega_0 = \omega_1/2$ is 0.0012 cm^3/s. This means that the maximum value of the flow during a central obstruction decreases by 99.7% for an occlusion of 75% of the cross-sectional area. (3) When a pressure gradient of the form of Eq. (21) with the new resonance frequency $2\omega_0 = \omega_2$ is imposed, the maximum value of the flow increases to 0.1022 cm^3/s. This implies a recovery of 26% of the flow.

For a dynamic of the Gaussian pulses, we obtained that the maximum value of the flow in a nonoccluded tube driven by a time-dependent pressure gradient with ω_1 is 0.1273 cm^3/s; the maximum value of the flow in a centrally occluded tube driven by exactly the same pressure gradient is 0.0002 cm^3/s. This means that the maximum value of the flow during a central obstruction decreases by 99.8% for an occlusion of 75% of the cross-sectional area; when a pressure gradient with the new resonance frequency ω_2 is imposed, the maximum value of the flow increases to 0.0258 cm^3/s. This implies a recovery of 20% of the flow.

Once more, we notice two things: that for a rectified sine signal the maximum value of the flow is larger in all cases, and that when it comes to the percent of flow that can be recovered, the dynamics does not make a dramatic difference, even though in this case the rectified sine signal gives a better flow recovery (26%) compared to the 20% given by a train of Gaussian pulses.

VII. DISCUSSION

According to our results, the overall effects of an obstruction are two. First, at low frequencies, the dynamic permeability is much smaller when there is an occlusion than when there is no occlusion. This can easily be understood because of the decrease in flow space and it has been understood for a very long time in the steady-state case. The second effect is a nontrivial one: the peak structure of the dynamic permeability is shifted toward the right-hand side, implying that the range of frequencies at which the real part of the dynamic permeability is large, happens at higher frequencies for occluded systems than for nonoccluded ones. We have also found that for the same obstructed area, the resonance frequency is much larger in a central occlusion than in a peripheral occlusion. This is due to the large friction that the fluid experiences flowing between the walls of the tube and the obstacle. Our results for the flow after an obstruction occurs, indicate that flow decrease is larger for central occlusions than for peripheral occlusions. This has been computed for the same obstructed cross-sectional area.

It is important to realize that in both types of occlusions, the value of the dynamic permeability at the resonance frequency, is as large as the value of the dynamic permeability at the resonance frequency of the nonoccluded case. For a one-mode pressure signal, this implies that by applying a pressure gradient containing the right frequency we can reach the value that the dynamic permeability had before the obstruction. Therefore, at the new frequency, the flow decrease due to the obstruction will then be given exclusively by the decrease in cross-sectional area. See Eq. (16).

For a general periodic pressure signal that contains many modes, the flow depends on an infinite sum involving the real and imaginary parts of the dynamic permeability at an infinite number of modes. Therefore, applying a periodic signal that contains the resonance frequency of the obstructed system, does not imply a full recovery of the flow; first, because the cross-sectional area available for flow has decreased and second, because the flow depends on the value of the permeability at different frequencies. However, as can be seen in Figs. 4 and 5, the flow can partially be recovered by applying a periodic pressure gradient containing the right frequency. The flow recovery is larger in the case of central occlusions.

Fluid flow in man-made systems is normally driven at either constant pressure gradient or constant flow. This means that for man-made systems, we generally encounter situations in which the fluid is moving in a steady state corresponding to zero frequency. On the contrary, on natural systems, fluids are often flowing at a particular frequency. For instance, in normal conditions blood is flowing in nonoccluded arteries at the frequency imposed by the heart. Amazingly, this natural frequency is very close to the resonance frequency of the system [2,3]. This suggests that blood in nonoccluded arteries is flowing at a frequency that minimizes the resistance to flow. Another interesting example is the one of mucus in bronchia at a frequency of cough (between 2 and 3 Hz for healthy individuals). Again amazingly, it turns out that the resonance frequency of and idealized system with the Maxwell parameters for mucus in bronchia is very close to the natural frequency of coughing [9]. What our results would suggest in this context is that, when an occlusion occurs in a system with a natural frequency that corresponds to the resonance frequency of the system, not only the flow decreases due to the decrease in flow space, but also because the resonance frequency of the occluded system changes while the frequency of flow, that is, the natural frequency of the system, remains unchanged. This would imply that in occluded systems the fluid is no longer flowing with the least possible resistance. The imposition of a pressure gradient at the new resonance frequency, that is, the resonance frequency of the occluded system, would cancel one of the two factors that provoke the dramatic decrease of flow during obstructions.

A word of caution is needed here since in many natural systems flow is rectified through a system of valves that has not been considered in the present work.

For both types of occlusions, we have considered that cylinders are rigid. However, many man-made and natural systems consist of flexible tubes. It has been shown [10] that the peak structure of the dynamic permeability remains qualitatively the same when longitudinal oscillations are considered. Important insight could be gained from the study of the effect of transverse waves.

We have studied two dynamics of the imposed pressure gradient and have obtained qualitatively similar results when it comes to percentage of flow recovery. Nevertheless a pressure gradient consisting of a rectified sine signal has given systematically larger values for the maximum flow than a pressure gradient consisting of a train of Gaussian pulses.

This implies that in order to maximize the flow magnitude for a particular system (fluid, confining media, and obstacle-shape), an optimization of the dynamics to be imposed would be necessary.

We have shown that the application of a pressure gradient with the correct frequency might help to partially recover the flow in obstructed systems. The theory presented here, might be useful to indicate which are the frequencies that would have to be imposed in order to achieve that. For instance, medical treatments based on the local imposition of certain frequencies to the flow, might help to improve local circulation.

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